# The Equation of Time - Early Days

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During the Portland NASS conference, Al Pratt asked questions about origins of the Equation of Time. I was unable to be at the conference, but here is my reply to Al's query.

## Note on Terminology

There is no direct translation of *Equation of Time* in Greek. The nearest related concept is νυχθημερον = nychthemeron = the duration of day and night *e.g.* from one solar noon to the next. In Arabic, *Equation of Time* is tacdīl al-ayyām bi layālayhā. In medieval Latin, *Equatio Dierum* = Equation of Days.

The term '*Equation of ……*' translates exactly as the '*Difference between observed and mean …….*'. It is used as in the Equation of Time, the Equation of the Center, the Equation of the Anomaly, the Equation of the Equinoxes, *etc*. The concept of mean days/years and the mean Sun was something well understood and extensively used in ancient and mediaeval astronomy. However, the ancient mean Sun is one that moves along the Ecliptic and thus is only indirectly associated with the modern mean Sun, which keeps our time and tracks the Equator.

## Introduction

The Greeks knew all about the Equation of Time - though not under that name and not for the purpose of telling the time as we understand it. Their knowledge related to astronomical, and in particular, lunar prediction, rather than time telling. There has been extensive study of the Equation of Time from the astronomical perspective from Greek to Renaissance times: see *References & Related Reading*.

In 1669, Christiaan Huygens (1629 - 1695) published the first 'modern' table of the Equation of Time in his "Instructions Concerning the Use of Pendulum-Watches for finding the Longitude at Sea". Thereafter the Equation of Time became the concern of time tellers and thus sundial makers. It was around this time that the term *Equatio Dierum* translated to *Equation of Time*.

## Brief History

The Babylonians were aware that the Sun's motion was not uniform. Between Virgo and Pisces, they reckoned that the Sun moved at 30° per lunar month, while in the other half of the year, it moved at 28;7.30° (= degrees/mins/secs in their sexagesimal system). The non-uniformity was certainly known to the Greek Callippus as early as 330 BCE.

Babylonian astronomers were measurers of events. Detailed daily records of astronomical events were made as early as 1500 BCE. They were primarily concerned with matching observed data to simple arithmetical rules so that predictions could be made. Heavenly phenomena were directly connected to omens - either good or bad. Omens were an important part of the ruler's toolkit of government. So good predictions were vital. (Nothing much has changed in that approach).

Greek astronomy began as a means to tell the seasons rather than the time: "When the Pleiades, daughters of Atlas, are rising, begin the harvest, the plowing when they set" - Hesiod: *Works and Days*. With the rise of reasoned thought, their approach to astronomy became concerned with why things happen and with 'saving the phenomena'. In this context, the word 'phenomenon' is used for an observed fact that must be described in terms of a given philosophy.

The philosophy used was that derived by Aristotle from Plato's *a priori* assumption that the Heavens were a perfect sphere and everything therein must be moving in circles at a uniform rate. This philosophy - coined as the "most regressive step in the history of science" - remained extant for almost two millennia, until Kepler's theories were finally accepted. Such was Aristotle's influence that it has been said that "Science, up to the Renaissance, consists in a series of footnotes to Aristotle" - Koestler: *Sleepwalkers*.

‘Saving the phenomena’ implied producing some kind of working model that could describe the movements of the “Wandering Stars” - the Planets and the Moon. The Fixed Stars were easy - they just obeyed Aristotle’s demands. The Planets, however, usually move in the direction of the Sun, but sometimes they go backwards - their so-called retrograde movement. This was hard to match to a philosophy of uniformity and perfection.

The first really viable general models that could explain the non-uniform motions of the Sun and the Planets were developed by Apollonius of Perga 262 - 190 BCE – famous amongst mathematicians as the geometer of conic sections. He developed two models:

- Epicyclic theory, which works well for sun, moon and planets, in which the heavenly body rotates uniformly around a small ‘epicycle’ circle, which is centered on the circumference of a large ‘deferent’ circle. The deferent is centered on the Earth. The epicycle center moves uniformly around the deferent. This fulfils the philosophical requirement. See Figure 1.
- Eccentric theory, which works well for the Sun, in which the heavenly body moves uniformly around a circle, but the Earth is not the center of the circle. This simpler model only somewhat fulfils the philosophical requirement. See Figure 2.

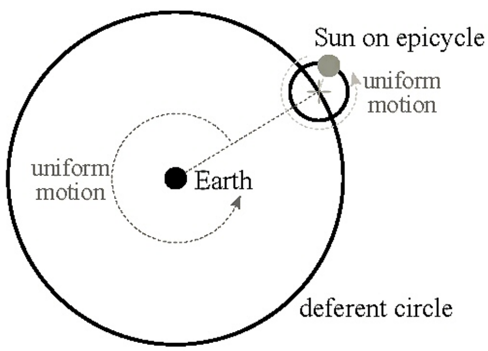



Fig. 1 Epicyclic Motion

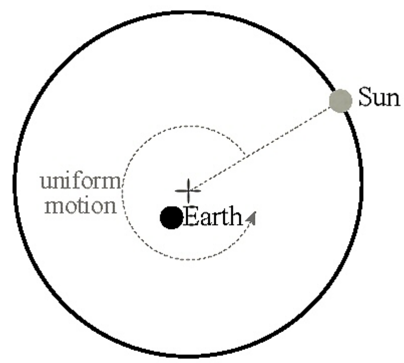



Fig. 2 Eccentric Motion

The two models, as shown, are mathematically the same. However, the Epicyclic model is more ‘versatile’ inasmuch as it is easy to describe the retrograde motion of the inferior planets and more complicated motions can be built by making an epicyclic circle to be the deferent of a yet smaller epicycle.

Hipparchos and the Sun’s Motion

After Alexander’s conquest of Mesopotamia, the extensive astronomical records of the Babylonian astronomers became available to the Greeks, and the next leap forward was made by the great Hipparchos of Rhodes, 190 -120 BCE, who invented trigonometry and who discovered Precession of the Equinoxes. Hipparchos is considered by many as the greatest ancient astronomer. He was not only a theoretician but one who backed his theories by measurement. He also invented the astrolabe and the equatorial ring to find the time of the equinoxes, but had nothing more sophisticated than a vertical gnomon with which to find the solstices, which is difficult since the Sun’s declination changes very slowly around those times.

Using many measurements of his own and by collating data that stretched back to Babylonian times, he set down the seasonal lengths as follows: Winter Solstice, 90 1/8 days to Vernal Equinox, then 94 ½ days to Summer Solstice, then 92 ½ days to Autumn Equinox, then 88 1/8 days to return to Winter Solstice - adding up to 365 ¼ days in total. Using these figures, he constructed an Eccentric model that adequately described the Sun’s motion. See Figure 3.

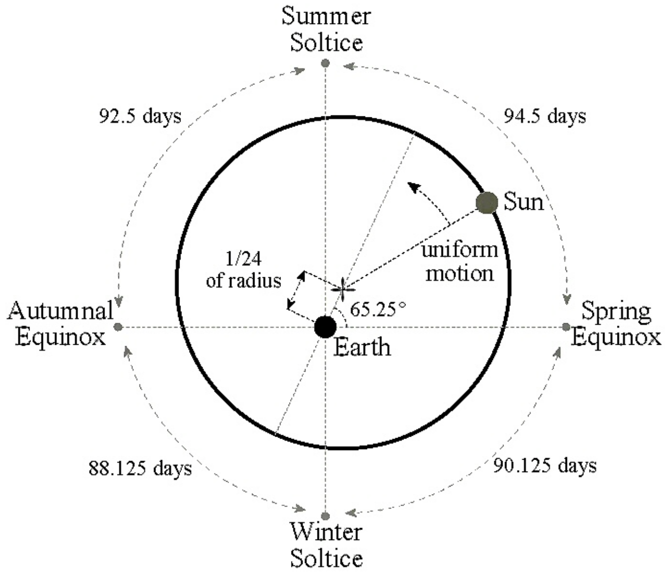



Fig. 3 Hipparchos' Eccentric Model of the Sun's Motion

Next, it appears that Geminus of Rhodes 110-40 BCE became aware that the time between one solar noon and the next - the nythchemeron - was not constant and that he considered this to be the result of the angle between the Equatorial & Ecliptic planes.

Ptolemy and Unequal Days

It was, however, the great Claudius Ptolemy of Alexandria, 85-165 AD, who first provided a comprehensive description of the phenomenon of 'unequal nythchemeron'. He provided the means to calculate the difference between nythchemeron and the length of a mean day – which is exactly what we understand by the daily change in the Equation of Time. Furthermore his description clearly delineated between the two elements that we now call the Eccentricity & the Obliquity components of the Equation of Time. Here are Ptolemy's own words on the subject…

Fig. 4 Claudius Ptolemy

"*On the inequality in the solar days*

*……it seems appropriate to add a brief discussion of the subject of the inequality of the solar day. A grasp of this topic is a necessary prerequisite, since the mean motions which we tabulate for each body are all arranged on the simple system of equal increments, as if all solar days were of equal length. However, it can be seen that this is not so. The revolution of the universe takes place uniformly about the poles of the equator. The more prominent ways of marking that revolution are by its return to the horizon, or to the meridian. Thus one revolution of the universe is, clearly, the return of a given point on the equator from some place on either the horizon or the meridian to the same place; and a solar day, simply defined, is the return of the sun from some point either on the horizon or on the meridian to the same point. On this definition, a mean solar day is the period comprising the passage of the 360 time-degrees of one revolution of the equator plus approximately 0;59 time-degrees, which is the amount of the mean motion of the sun during that period; and an anomalistic solar day is the period comprising the passage of the 360 time-degrees of one revolution of the equator plus that stretch of the equator which rises with, or crosses the meridian with, the anomalistic motion of the sun in that period. This additional stretch of the equator, beyond the 360 time-degrees, which crosses the horizon or meridian cannot be a constant, for two reasons: firstly, because of the sun's apparent anomaly; and secondly, because equal sections of the ecliptic do not cross either the horizon or the meridian in equal times. Neither of these effects causes a perceptible difference between the mean and the anomalistic return for a single solar day, but the accumulated difference over a number of solar days is quite noticeable……*" *Almagest III 9*, translated by G.J. Toomer.

The 'apparent anomaly' is the difference between the sun's actual movement along the ecliptic compared with its mean movement: this is the Eccentricity component of the Equation of Time. The 'equal sections of the ecliptic do no cross either the horizon or the meridian in equal times' describes the Obliquity component of the Equation.

<u>With only a Sundial and a Water Clock to tell the Time, Why bother with this Theory?</u>

Again, quoting Ptolemy's own words…

*"... to neglect a difference (in the nythchemeron) of this order would, perhaps, produce no perceptible error in the computation of phenomena associated with the sun and other planets; but in the case of the moon, since its speed is so great, the resulting error could no longer be overlooked, since it could amount to 3/5 of a degree…*"

The ability to foretell solar, planetary and in particular lunar events was of great interest to both astronomers and their political paymasters. It gave them the edge that knowledge provides over the common people whose understanding reached only as far as the realms of Astrology. With observations spanning many centuries and events timed by either a gnomon or by the rising and setting of the fixed stars, they achieved more precision than one might expect.

<u>How Good was the Theory?</u>

Having mastered the theory, how good were Ptolemy's calculations? Bearing in mind that he used the sexagesimal numbering system and used 'chords' rather than our familiar sines & cosines, his calculations are somewhat opaque for the lay person to follow. However, R.H. van Gent of Utrecht University has handily provided a web-based calculator that embodies all of Ptolemy's calculations. Figure 5 shows the difference between the Equation of Time derived therefrom and that calculated using the Jet Propulsion Lab's Horizons program for the year 150 AD. The similarity in shape is remarkable, and the error has a standard deviation of 1 ¼ mins.

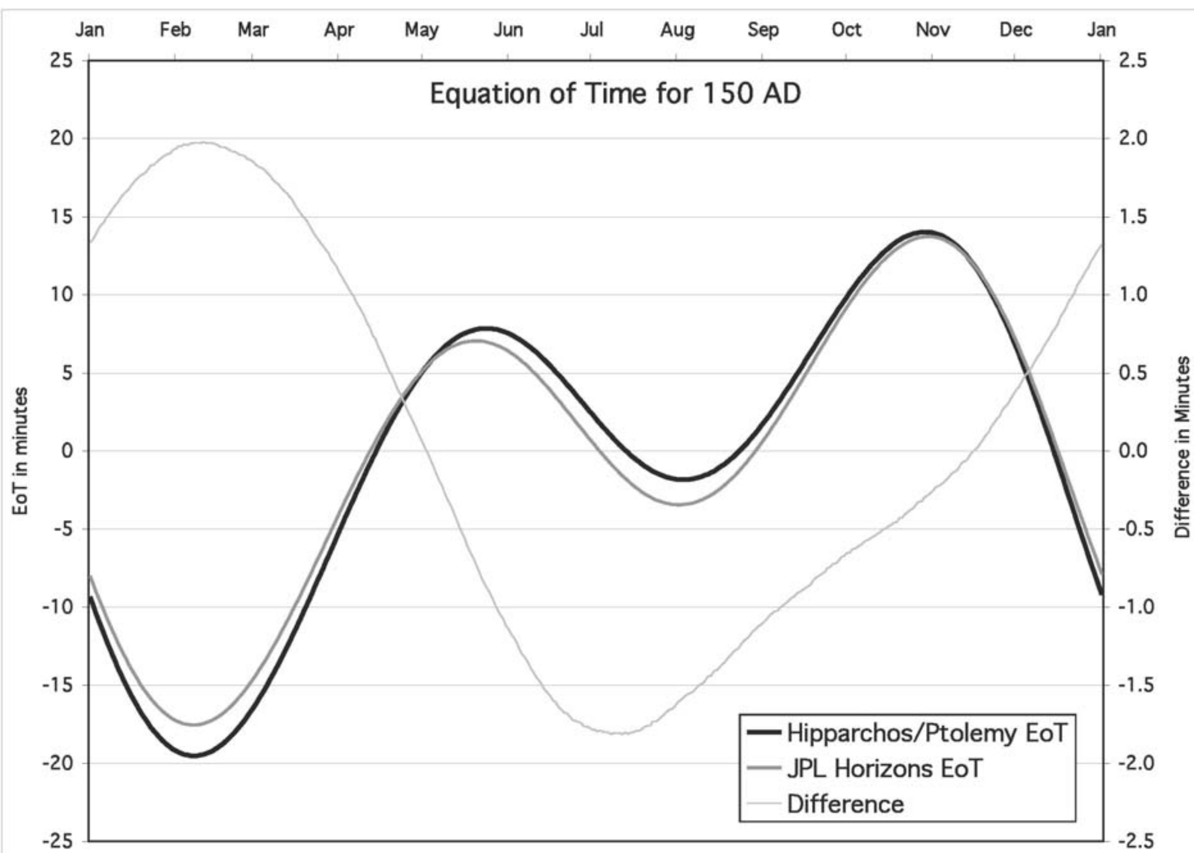



Fig. 5 Ptolemy's own Equation of Time

Note. Ptolemy started his whole cosmology at Noon in Alexandria on Day 1 of the Month Thoth of the Era Nabonassar = 26th February 746 BCE. From this date he considered that there was a continuous reliable record of astronomical events covering both the Babylonian and Greek eras. At that time, his Equation of Time was zero. In 150 AD, 900 years later, his calculated values had all shifted so that they spanned between -1 and 28 minutes. To make the comparison between modern and ancient calculations, Ptolemy's figures have been linearly shifted to make his annual average Equation of Time the same as the annual average Equation calculated by the Horizons program.

Bearing in mind that the calculations illustrated in Figure 5 were made using measurements that were made at best with a gnomon, alidade, quadrant and equinoctial ring, Figure 6 shows how the accuracy is improved if we substitute the modern parameters for the times between solstices and equinoxes and the value of obliquity. On the one hand, one may be amazed at how good the Greek theory was. But, on the other hand, the degree of accuracy obtained reflects our tiny orbital eccentricity and the fact that Ptolemy's method to obtain the Obliquity component was no better or worse than that found using modern trigonometry.

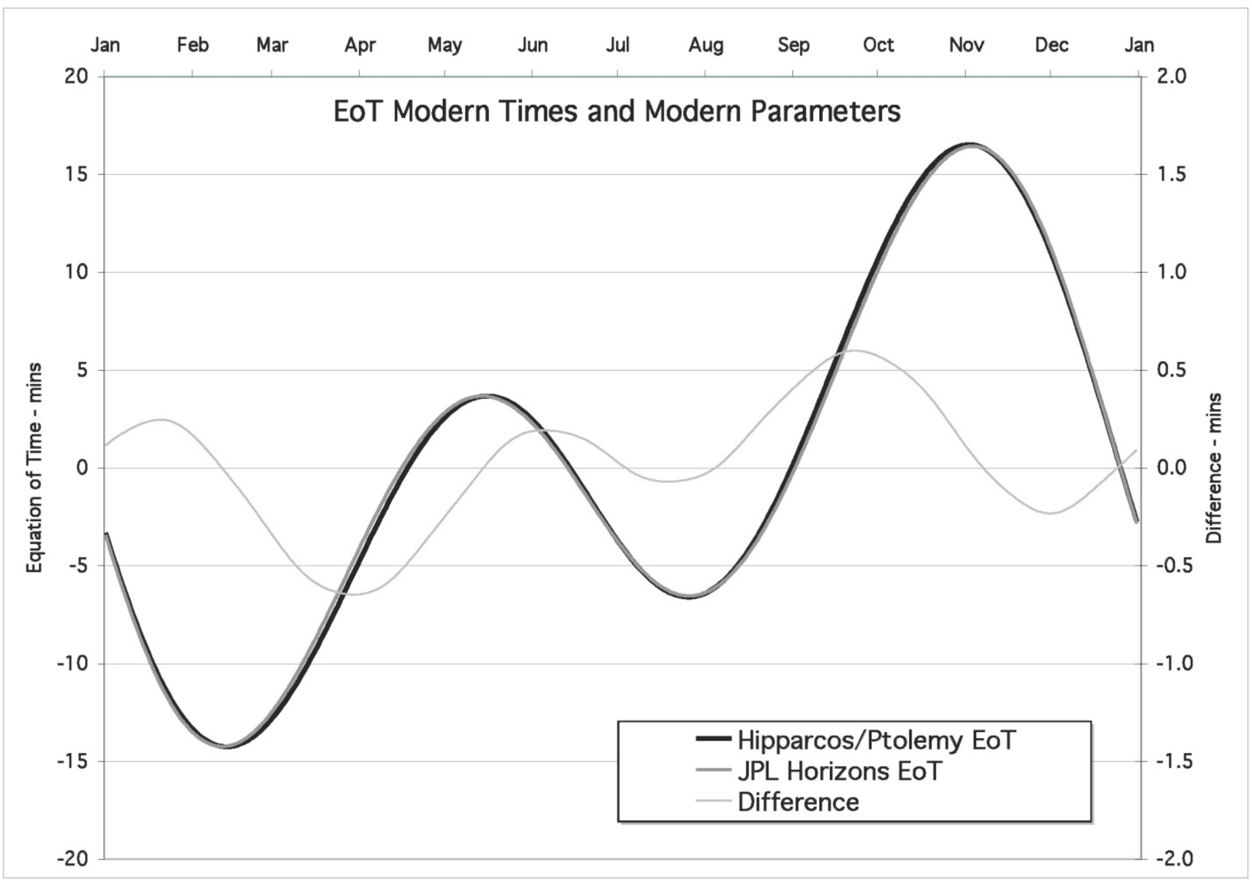



Fig. 6 Ptolemy's method, using modern parameters

Ptolemy's Legacy

Ptolemy was a man of towering intellect and influence, on a par with Euclid, Newton, Darwin and Einstein. His works are recorded in a number of books – one on *Optics* (lost), *Tetrabiblos* (which is on Astrology), *Geographica* (on map making and with maps), *Handy Tables* (the original Astronomical Almanac – which tabulates the Equation of Time - see Fig. 7 - and much else) and *Mathematical Syntaxis* (his major contribution to astronomical theory).

This last book was so important that the Arab astronomers re-named it Al-mjsty (the Greatest) from which its current title *The Almagest* is derived. It was translated from its original Greek into Syriac and then into a number of Arabic versions as part of the great Translation Movement in the 8th century. From Arabic, it was translated to Latin in Toledo in 1175. The work entered the mainstream of western astronomy with the commentary by Peurbach and Regimontanus in 1475, and its printing in Venice in 1515. The earliest known version of the Almagest is in the Vatican library. It is in C9 Greek and presumably from the libraries of Constantinople. There is little doubt, however, that the main influence of the Almagest in Europe came via the Arabic translations. The Almagest was the standard textbook on astronomy until the end of the Renaissance – a period of time only superseded in the technical world by Euclid on geometry and Galen on medicine.

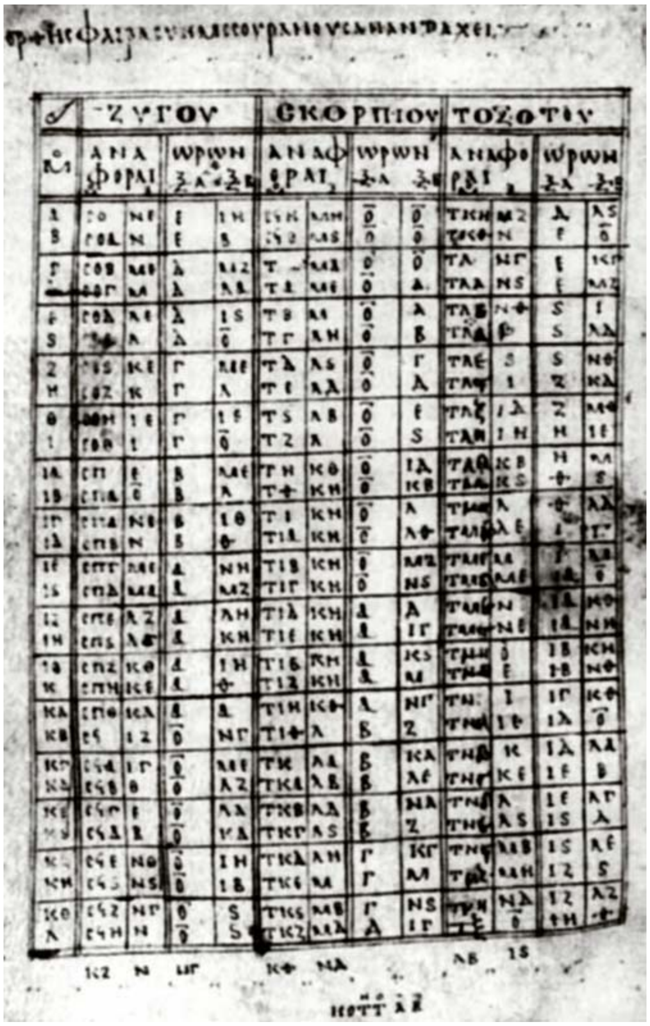


Fig. 7 Ptolemy's Handy Tables. A fragment of Leiden BPG 78 showing the Equation of Time

It is strongly recommend that the reader find a copy of Ptolemy's *Almagest* by G.J Toomer, Princeton Press, which is a recent and ultimately authoritative translation. Just a quick look at the text and the diagrams will demonstrate the complexity and intellectual scale of the man.

Why did the Legacy Last so Long

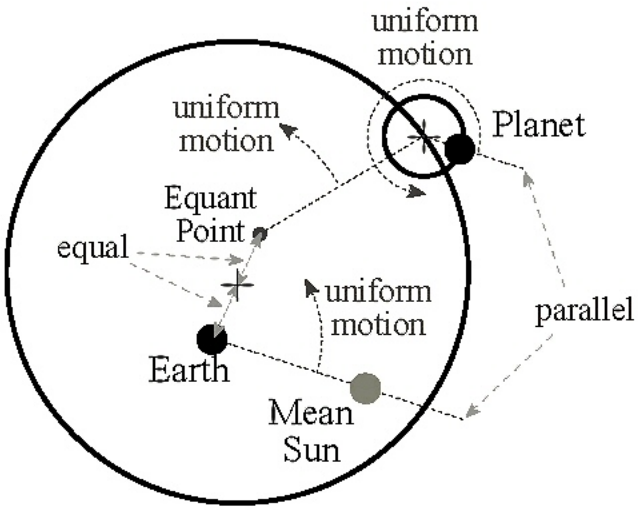
Although, there was much work on the Equation of Time by Islamic and Mediaeval astronomers - for example: al-Battānī (C9), al-Khwārizmī (C9), Kūshyār (C10), al-Kāshī (C14), there was nothing more substantial than improvements to the computation of solar longitude and the value of obliquity.

There were many reasons for the longevity of Ptolemy's legacy – the dark ages, the burning of the Alexandrian library, the great schism, scholastic dogmatism towards Aristotle's precepts and the neatness with which Aristotle's cosmology fitted the requirements of both Christian and Muslim faiths. However, there were two technical reasons why all of Ptolemy's works survived so long.

Firstly, Ptolemy's theory for the planets (if not for the Sun) was technically very demanding to understand. When Alphonso X The Wise of Castile (1221 - 1284 AD), who was a great patron of astronomers, was being taught the Ptolemaic system, he is quoted as saying...

"If the Lord Almighty had consulted me before embarking upon the Creation, I should have recommended something simpler."

Secondly, to 'save the phenomenon' for moon and planets - *i.e.* to fit theory to observation - Ptolemy found he needed to move the center of the deferent away from the Earth by a small amount and to move the center of epicyclic uniform motion still further from the earth by an equal small amount to the Equant Point (Punctum Aequans): see Fig. 8. This imaginary point deeply offended both Islamic and mediaeval western astronomers since it flew directly in the face of the Aristotelean philosophical requirements. As a result, the astronomers - notably, the Persian al-Tūsī (C13), the Syrians al-'Urdi (C13) and Ibn al-Shatir (C14) and Nicolaus Copernicus (C16) - spent an inordinate amount of intellectual energy trying to eradicate the Equant Point by adding epicycles on epicycles: see Ref. 13. With the wonder of hindsight, we now see the Equant Point as equivalent to the second of the foci of Kepler's ellipses.

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12. Kevin Lee(?), University of Nebraska-Lincoln: http://astro.unl.edu/naap/ssm/animations/ptolemaic.swf. A good animation to show the Equant Point.
13. Dennis Duke: http://people.sc.fsu.edu/~dduke/arabmars.html. A good animation of Islamic attempts to eradicate the Equant Point.

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*[Editor's Note: Those readers who take Kevin up on his invitation to examine G.J. Toomer's translation of Ptolemy's Almagest are alerted to an interesting note 53 on page 449. In this note, Toomer refers to a contribution to Ptolemaic theory by Fred Sawyer in his graduate school days as "an acute critique of the method employed by Ptolemy for determining the apsidal line of the inner planets."]*